Higher Spins from Nonlinear Realizations of $OSp(1|8)$

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Abstract

We exhibit surprising relations between higher spin theory and nonlinear realizations of the supergroup $OSp(1|8)$, a minimal superconformal extension of $N = 1$, $4D$ supersymmetry with tensorial charges. We construct a realization of $OSp(1|8)$ on the coset supermanifold $OSp(1|8)/SL(4,R)$ which involves the tensorial superspace $R^{(10|4)}$ and Goldstone superfields given on it. The covariant superfield equation encompassing the component ones for all integer and half-integer massless higher spins amounts to the vanishing of covariant spinor derivatives of the suitable Goldstone superfields, and, via Maurer-Cartan equations, to the vanishing of $SL(4,R)$ supercurvature in odd directions of $R^{(10|4)}$. Aiming at higher spin extension of the Ogievetsky-Sokatchev formulation of $N = 1$ supergravity, we generalize the notion of $N = 1$ chirality and construct first examples of invariant superfield actions involving a non-trivial interaction. Some other potential implications of $OSp(1|8)$ in the proposed setting are briefly outlined.

1 Introduction

Since the seminal papers by Fradkin and Vasiliev [1], the theory of higher spin fields is under intensive development (see e.g. [2, 3] and refs. therein). Nowadays it attracts vast attention due to its profound relations to string theory and AdS/CFT hypothesis. A concise and suggestive way to deal with higher spins is to allow for the dependence of fields on additional coordinates, in particular the tensorial ones, generated by tensorial charges [4]–[8]. In $4D$, in order to justify geometrically the appearance of the tensorial charges, one should extend the standard supersymmetry algebra to $4D$ counterpart of M-theory algebra [9, 10] and look for the corresponding dynamical models.

Developing the conjecture of Fronsdal [4], Vasiliev has shown in [7] that the free $4D$ higher spin field theory can be described by the pair of bosonic and fermionic fields $b(Y), f_{\hat{\alpha}}(Y)$ ($\hat{\alpha} = 1, 2, 3, 4$) defined on ten-dimensional real tensorial space

$$Y^{\hat{\alpha}\hat{\beta}} = Y^{\hat{\beta}\hat{\alpha}} = \frac{1}{2}\, x^m (\gamma_m)^{\hat{\alpha}\hat{\beta}} + \frac{1}{4}\, y^{[m\,n]}(\gamma_{[m\,n]})^{\hat{\alpha}\hat{\beta}} \tag{1.1}$$

where x^m are Minkowski space coordinates. A nice superfield form of these equations, in the tensorial superspace $R^{(10|4)} = (Y^{\hat{\alpha}\hat{\beta}}, \theta^{\hat{\alpha}}) \equiv \tilde{Y}^M$, was recently suggested in [8].[1]

Keeping in mind the distinguished role of tensorial (super)spaces in the higher spin theory, it is of urgent importance to get better insights into their geometry, as well as to work out the superfield methods of the appropriate model-building, including construction of the appropriate off-shell superfield actions for higher spins.

The basic aim of the present paper is to show that an adequate framework for addressing these and related physically motivated problems is provided by nonlinear realizations of the supergroup $OSp(1|8)$ which is a minimal superconformal extension of $N = 1$, $4D$ supersymmetry with tensorial charges [11, 12].

We construct a nonlinear realization of $OSp(1|8)$ in the supercoset

$$\widetilde{\mathcal{K}} = \frac{OSp(1|8)}{SL(4, R)} \tag{1.2}$$

which is the direct analog of the well known coset of the standard $4D$, $N = 1$ super-conformal group [2]

$$\mathcal{K} = \frac{SU(2, 2|1)}{SL(2, C) \times U(1)}\,. \tag{1.3}$$

The supercoset (1.2) involves as its parameters the $R^{(10|4)}$ superspace coordinates and some Goldstone superfields defined on this superspace. Our main tool is the formalism of left-covariant Cartan one-forms, supplemented with covariant constraints on the Goldstone superfields covariant derivatives. These constraints contain, as an essential part, the inverse Higgs conditions [15] allowing one to algebraically eliminate all Goldstone superfields in terms of single superfield associated with the dilatation generator. Simultaneously, they imply the correct dynamical equations for this basic superfield which coincide, after some field redefinition, with the equation given in [8]. Thus one of the novel points of our nonlinear realization approach is that the basic superfield

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[1]This superfield equation can be also recovered as a result of quantization of free twistor superparticle propagating in $R^{(10|4)}$ [6].

[2]Nonlinear realizations of $SU(2, 2|1)$ in such a coset were considered in [13] as a natural extension of standard nonlinear realizations of the conformal group $SO(2, 4)$ [14].

encompassing all higher spins appears as a parameter of the supercoset (1.2). Another point is the new geometric interpretation of this equation. It proves to be the condition of vanishing of the covariant spinor derivatives of the Goldstone superfields associated with the generators of dilatations and conformal supersymmetry. Via Maurer-Cartan equations, these conditions lead to the vanishing of the $SL(4, R)$ supercurvature along the pure odd directions in $R^{(10|4)}$.

Besides offering a novel view on the free higher spin dynamics in the superspace $R^{(10|4)}$, the nonlinear realizations approach allows one to find out another interesting coset supermanifold of $OSp(1|8)$, which is a generalization of the chiral $N = 1, 4D$ superspace. The latter is known to play the fundamental role in ordinary $N = 1$ supersymmetric theories, so its tensorial counterpart is expected to have similar implications in higher spin $N = 1$ theories. It is $C^{(11|2)} = (x_L^{\alpha\dot\beta}, z_L^{\alpha\beta}, f_L^{\alpha\dot\beta}, \theta_L^\alpha)$ involving, besides complex Minkowski coordinate and chiral half of Grassmann coordinates[3], also the holomorphic half of the tensorial coordinates $z_L^{\alpha\beta}$ and the extra complex coordinates $f_L^{\alpha\dot\beta}$ which provide a holomorphic parametrization of the 'harmonic' coset $SL(4, R)/GL(2, C)$. We define corresponding generalized chiral superfields and construct for them two $OSp(1|8)$ invariant off-shell actions which are analogs of the kinetic and potential terms of the ordinary chiral $N = 1$ superfields.

The problem of extension of the nonlinear realizations framework to the non-flat (in particular, corresponding to AdS structure) tensorial (super)spaces is now under investigation. We hope that our nonlinear realization approach will prove useful in constructing off-shell actions for higher spin fields, as well as for better understanding of the structure of the higher-spin extensions of superfield $N = 1$ supergravity [8]. The generalized chirality seems to be especially promising in the latter aspect, recalling the Ogievetsky-Sokatchev formulation of $N = 1, 4D$ supergravity [16].

2 $OSp(1|8)$ as a generalized superconformal group

The even (bosonic) sector of the superalgebra $osp(1|8)$ is the generalized $4D$ conformal algebra $sp(8)$ which is a closure of the standard conformal algebra $so(2, 4)$ and the algebra $sl(4, R)$.

The algebra $so(2, 4) \simeq su(2, 2)$ is spanned by the generators $(L_{\alpha\dot\beta}, \overline{L}_{\dot\alpha\dot\beta}, P_{\alpha\dot\beta}, K_{\alpha\dot\beta}, D)$

$$[P_{\alpha\dot\beta}, P_{\gamma\dot\delta}] = [K_{\alpha\dot\beta}, K_{\gamma\dot\delta}] = 0 \tag{2.1a}$$

$$[P_{\alpha\dot\beta}, K_{\rho\dot\lambda}] = \frac{1}{2}\left(\epsilon_{\alpha\rho}\bar{L}_{\dot\beta\dot\lambda} - \epsilon_{\dot\beta\dot\lambda}L_{\alpha\rho}\right) - i\epsilon_{\alpha\rho}\epsilon_{\dot\beta\dot\lambda}D, \tag{2.1b}$$

$$[L_{\alpha\beta}, L_{\rho\lambda}] = \epsilon_{\alpha\rho}L_{\beta\lambda} + \epsilon_{\beta\rho}L_{\alpha\lambda} + \epsilon_{\alpha\lambda}L_{\beta\rho} + \epsilon_{\beta\lambda}L_{\rho\alpha}, \tag{2.1c}$$

$$[L_{\alpha\beta}, P_{\rho\dot\rho}] = \epsilon_{\alpha\rho}P_{\beta\dot\rho} + \epsilon_{\beta\rho}P_{\alpha\dot\rho}, \quad [L_{\alpha\beta}, K_{\rho\dot\rho}] = \epsilon_{\alpha\rho}K_{\beta\dot\rho} + \epsilon_{\beta\rho}K_{\alpha\dot\rho}, \tag{2.1d}$$

$$[D, P_{\alpha\dot\alpha}] = iP_{\alpha\dot\alpha}, \quad [D, K_{\alpha\dot\alpha}] = -iK_{\alpha\dot\alpha}. \tag{2.1e}$$

The rest of non-vanishing commutators can be obtained by complex conjugation.

The algebra $sl(4, R)$ is spanned by the generators $(L_{\alpha\beta}, \overline{L}_{\dot\alpha\dot\beta}, A, F_{\alpha\dot\beta}, \overline{F}_{\alpha\dot\beta})$. The extra

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[3]We use here $4D$ Weyl spinor notation.

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generators A, $F_{\rho\dot\tau}$, $\overline{F}_{\rho\dot\tau} \equiv (F_{\tau\dot\rho})^*$ satisfy the relations

$$[F_{\alpha\dot\beta}, \overline{F}_{\beta\dot\nu}] = 2\epsilon_{\alpha\beta}\epsilon_{\dot\beta\dot\nu}A + 2\left(\epsilon_{\alpha\beta}\bar{L}_{\dot\beta\dot\nu} - \epsilon_{\dot\beta\dot\nu}L_{\alpha\beta}\right), \tag{2.2a}$$

$$[F_{\alpha\dot\alpha}, F_{\beta\dot\nu}] = [\overline{F}_{\alpha\dot\alpha}, \overline{F}_{\beta\dot\nu}] = 0, \tag{2.2b}$$

$$[A, F_{\alpha\dot\beta}] = 2F_{\alpha\dot\beta}, \quad [A, \overline{F}_{\alpha\dot\beta}] = -2\overline{F}_{\alpha\dot\beta}. \tag{2.2c}$$

The generalized $4D$ conformal algebra $sp(8)$ is a closure of the algebras $so(2,4)$ and $sl(4,R)$. It is obtained by adding to the generators of $sl(4,R)$ and the vectorial Abelian translation generators $(P_{\alpha\dot\beta}, K_{\alpha\dot\beta})$ the following additional 12 Abelian generators

- $(Z_{\alpha\beta}, \overline{Z}_{\dot\alpha\dot\beta})$ describing six standard tensorial translations

- $(\widetilde{Z}_{\alpha\beta}, \overline{\widetilde{Z}}_{\dot\alpha\dot\beta})$ describing six conformal tensorial translations.

They satisfy the following commutation relations:

$$[Z_{\alpha\beta}, \widetilde{Z}_{\rho\lambda}] = \frac{1}{2}\left(\epsilon_{\alpha\rho}L_{\beta\lambda} + \epsilon_{\beta\rho}L_{\alpha\lambda} + \epsilon_{\alpha\lambda}L_{\beta\rho} + \epsilon_{\beta\lambda}L_{\alpha\rho}\right)$$
$$+ \left(\epsilon_{\alpha\rho}\epsilon_{\beta\lambda} + \epsilon_{\beta\rho}\epsilon_{\alpha\lambda}\right)\left(iD - \frac{1}{2}A\right), \tag{2.3a}$$

$$[P_{\alpha\dot\beta}, \widetilde{Z}_{\rho\beta}] = \frac{1}{2}\left(\epsilon_{\alpha\rho}\overline{F}_{\beta\dot\beta} + \epsilon_{\alpha\beta}\overline{F}_{\rho\dot\beta}\right), [K_{\alpha\dot\beta}, Z_{\rho\beta}] = \frac{1}{2}\left(\epsilon_{\alpha\rho}F_{\beta\dot\beta} + \epsilon_{\alpha\beta}F_{\rho\dot\beta}\right), \tag{2.3b}$$

$$[P_{\alpha\dot\alpha}, F_{\beta\dot\beta}] = -2\epsilon_{\dot\alpha\dot\beta}Z_{\alpha\beta}, [Z_{\alpha\beta}, F_{\rho\dot\nu}] = 0, [Z_{\alpha\rho}, \bar{F}_{\gamma\dot\nu}] = 2\left(\epsilon_{\alpha\gamma}P_{\rho\dot\nu} + \epsilon_{\rho\gamma}P_{\alpha\dot\nu}\right), \tag{2.3c}$$

$$[K_{\alpha\dot\alpha}, F_{\beta\dot\beta}] = 2\epsilon_{\alpha\beta}\overline{\widetilde{Z}}_{\dot\alpha\dot\beta}, [\widetilde{Z}_{\alpha\beta}, \overline{F}_{\rho\dot\nu}] = 0, [\widetilde{Z}_{\alpha\rho}, F_{\gamma\dot\nu}] = 2\left(\epsilon_{\alpha\gamma}K_{\rho\dot\nu} + \epsilon_{\rho\gamma}K_{\alpha\dot\nu}\right), \tag{2.3d}$$

$$[A, Z_{\alpha\rho}] = 2Z_{\alpha\rho}, \quad [D, Z_{\alpha\rho}] = iZ_{\alpha\rho}, \tag{2.3e}$$

$$[A, \widetilde{Z}_{\alpha\rho}] = -2\widetilde{Z}_{\alpha\rho}, \quad [D, \widetilde{Z}_{\alpha\rho}] = -i\widetilde{Z}_{\alpha\rho}. \tag{2.3f}$$

The remaining commutators are either vanishing, or can be obtained by complex conjugation from the above ones, taking into account the rules $\overline{A} = A, \overline{D} = D$.

The odd (fermionic) sector of $osp(1|8)$ involves $N = 1$ super Poincaré generators $Q_\alpha, \bar{Q}_{\dot\alpha}$ and the generators $S_\alpha, \bar{S}_{\dot\alpha}$ of conformal supersymmetry.[4] The basic algebraic relations look as follows

i) basic superalgebra relations

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2P_{\alpha\dot\alpha}, \quad \{Q_\alpha, Q_\beta\} = 2Z_{\alpha\beta}, \quad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 2\overline{Z}_{\dot\alpha\dot\beta},$$

$$\{S_\alpha, \bar{S}_{\dot\alpha}\} = 2K_{\alpha\dot\alpha}, \quad \{S_\alpha, S_\beta\} = 2\widetilde{Z}_{\alpha\beta}, \quad \{\bar{S}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = 2\overline{\widetilde{Z}}_{\dot\alpha\dot\beta},$$

$$\{Q_\alpha, \bar{S}_{\dot\beta}\} = F_{\alpha\dot\beta}, \quad \{S_\alpha, \bar{Q}_{\dot\beta}\} = \bar{F}_{\alpha\dot\beta},$$

$$\{Q_\alpha, S_\beta\} = \epsilon_{\alpha\beta}\left(iD - \frac{1}{2}A\right) + L_{\alpha\beta},$$

$$\{\bar{Q}_{\dot\alpha}, \bar{S}_{\dot\beta}\} = -\epsilon_{\dot\alpha\dot\beta}\left(iD + \frac{1}{2}A\right) + \bar{L}_{\dot\alpha\dot\beta}, \tag{2.4a}$$

[4]Standard $N = 1$, $D = 4$ superconformal symmetry $su(2,2|1)$ is not a subalgebra of $osp(1|8)$. These two superalgebras describe two different superextensions of $4D$ conformal symmetry (see discussion in [17]).

ii) covariance relations for supercharges

$$[A, Q_\alpha] = Q_\alpha \,, \ \ [A, \bar{Q}_{\dot\alpha}] = -\bar{Q}_{\dot\alpha} \,, \ \ [D, Q_\alpha] = \frac{i}{2} Q_\alpha \,, \ \ [D, \bar{Q}_{\dot\alpha}] = \frac{i}{2} \bar{Q}_{\dot\alpha} \,,$$

$$[A, S_\alpha] = -S_\alpha \,, \ \ [A, \bar{S}_{\dot\alpha}] = \bar{S}_{\dot\alpha} \,, \ \ [D, S_\alpha] = -\frac{i}{2} S_\alpha \,, \ \ [D, \bar{S}_{\dot\alpha}] = -\frac{i}{2} \bar{S}_{\dot\alpha} \,,$$

$$[Q_\alpha, F_{\rho\dot\beta}] = 0 \,, \ \ [Q_\alpha, \overline{F}_{\rho\dot\beta}] = 2\epsilon_{\alpha\rho} \bar{Q}_{\dot\beta} \,, \ \ [S_\alpha, \overline{F}_{\rho\dot\beta}] = 0 \,, \ \ [S_\alpha, F_{\rho\dot\beta}] = 2\epsilon_{\alpha\rho} \bar{S}_{\dot\beta} \,,$$

$$[\widetilde{Z}_{\alpha\beta}, Q_\rho] = \epsilon_{\alpha\rho} S_\beta + \epsilon_{\beta\rho} S_\alpha \,, \ \ \ [\widetilde{\overline{Z}}_{\dot\alpha\dot\beta}, Q_\rho] = 0 \,,$$

$$[Z_{\alpha\beta}, S_\rho] = \epsilon_{\alpha\rho} Q_\beta + \epsilon_{\beta\rho} Q_\alpha \,, \ \ \ [\overline{Z}_{\dot\alpha\dot\beta}, S_\rho] = 0 \,,$$

$$[P_{\alpha\dot\alpha}, S_\rho] = \epsilon_{\alpha\rho} \bar{Q}_{\dot\alpha} \,, \ \ [K_{\alpha\dot\alpha}, Q_\rho] = \epsilon_{\alpha\rho} \bar{S}_{\dot\alpha} \,. \tag{2.4b}$$

All other (anti)commutators vanish except the complex conjugates of (2.4b).

3 Nonlinear realizations of $OSp(1|8)$

Before constructing nonlinear realization of $OSp(1|8)$ in the supercoset (1.2), we consider the bosonic limit of this realization. Namely, we consider $Sp(8) \subset OSp(1|8)$ and construct an $Sp(8)$ analog of the nonlinear realization of ordinary conformal group $SO(2,4)$ in the coset $SO(2,4)/SO(1,3)$ [14]. It corresponds to the choice of the coset $K = Sp(8)/SL(4, R)$ spanned by the following generators

$$K : \ (P_{\alpha\dot\beta}, Z_{\alpha\beta}, \overline{Z}_{\dot\alpha\dot\beta}, K_{\alpha\dot\beta}, \widetilde{Z}_{\alpha\beta}, \widetilde{\overline{Z}}_{\dot\alpha\dot\beta}, D) \,. \tag{3.1}$$

We represent the coset K by the following element of $Sp(8)$

$$g = e^{i(x \cdot P + z \cdot Z)} \, e^{i\phi D} \, e^{i(k \cdot K + t \cdot \widetilde{Z})} \tag{3.2}$$

where

$$x \cdot P = x^{\alpha\dot\alpha} P_{\alpha\dot\alpha} \,, \ k \cdot K = k^{\alpha\dot\alpha} K_{\alpha\dot\alpha} \,, \ z \cdot Z = z^{\alpha\beta} Z_{\alpha\beta} + \bar{z}^{\dot\alpha\dot\beta} \overline{Z}_{\dot\alpha\dot\beta} \,, \ \text{etc} \,. \tag{3.3}$$

The group $Sp(8)$ acts on this element from the left, producing the corresponding transformation of the coset parameters.

According to the general rules of nonlinear realizations, we are led to consider $(x^{\alpha\dot\alpha}, z^{\alpha\beta}, \bar{z}^{\dot\alpha\dot\beta}) \equiv Y^{\hat\alpha\hat\beta}$ as coordinates and the rest of the coset parameters as Goldstone fields living on this extended ten-dimensional space. The basic objects of the considered nonlinear realizations framework are the left-covariant Cartan one-forms [5]

$$g^{-1}dg = i(\,\omega_P \cdot P + \omega_Z \cdot Z + \omega_D D + \omega_K \cdot K + \omega_{\widetilde{Z}} \cdot \widetilde{Z} + \omega_F \cdot F + \omega_L \cdot L + \omega_A A\,). \tag{3.4}$$

[5]Cartan forms for the supergroups $OSp(1|n)$ and $OSp(N|n)$ treated as curved versions of tensorial superspaces, with AdS subspaces instead of the Minkowski ones, were constructed in [18]- [21]; for $n = 8$ see [13]. The new input of our construction is that we treat $OSp(1|8)$ as a spontaneously broken symmetry realized in the supercoset (1.2) in which the ten-dimensional $4D$ tensorial superspace forms a coordinate subspace, while other coset parameters are Goldstone superfields with suitable constraints.

Explicitly, the forms necessary for our consideration are

$$\omega_P^{\alpha\dot\alpha} = e^\phi \, dx^{\alpha\dot\alpha}\,, \ \ \omega_Z^{\alpha\beta} = e^\phi \, dz^{\alpha\beta}\,, \ \ \omega_D = d\phi + e^\phi \,(dx \cdot k) - 2e^\phi \,(dz \cdot t)\,,$$

$$\omega_L^{\alpha\beta} = ie^\phi \left[2dz^{\gamma(\alpha}t_\gamma^{\beta)} - \frac{1}{2}dx^{(\alpha\dot\rho}k_{\dot\rho}^{\beta)} \right]\,, \ \ \omega_A = ie^\phi \left[d\bar{z}^{\dot\alpha\dot\beta}\bar{t}_{\dot\alpha\dot\beta} - dz^{\alpha\beta}t_{\alpha\beta} \right]\,,$$

$$\omega_F^{\alpha\dot\beta} = ie^\phi \left[dz^{\alpha\gamma}k_\gamma^{\dot\beta} - dx^{\alpha\dot\rho}\bar{t}_{\dot\rho}^{\dot\beta} \right]\,, \ \ \ \omega_{\bar{L}}^{\dot\alpha\dot\beta} = \overline{\omega_L^{\alpha\beta}}\,, \ \ \ \omega_{\bar{F}}^{\dot\alpha\dot\beta} = \overline{\omega_F^{\alpha\dot\beta}}\,. \tag{3.5}$$

Now we are prepared to consider a nonlinear realization of the supergroup $OSp(1|8)$ in the coset $\widetilde{\mathcal{K}}$ (1.2). For this purpose we add to the previous coset generators the spinor generators, $Q_\alpha, \bar{Q}_{\dot\alpha}$ and $S_\alpha, \bar{S}_{\dot\alpha}$. Correspondingly, we introduce new coset coordinates, the spinor coordinates $\theta^\alpha, \bar{\theta}^{\dot\alpha}$ extending the previous bosonic space $Y^{\hat\alpha\hat\beta} \equiv (x^{\alpha\dot\beta}, z^{\alpha\beta}, \bar{z}^{\dot\alpha\dot\beta})$ to the superspace $\widetilde{Y}^M \equiv (Y^{\hat\alpha\hat\beta}, \theta^\alpha, \bar{\theta}^{\dot\alpha})$ and the spinor Goldstone superfields $\psi^\alpha(\widetilde{Y}), \bar{\psi}^{\dot\alpha}(\widetilde{Y})$. We parametrize the supercoset elements as follows [6]

$$G = e^{i(\theta Q + \bar{\theta}\bar{Q})} \, g \, e^{i(\psi S + \bar{\psi}\bar{S})}\,, \tag{3.6}$$

where g is the same bosonic coset element as defined in (3.2), with all parameter-fields now being superfields on the superspace $\widetilde{Y}^M$. The Cartan forms are defined by:

$$
\begin{aligned}
G^{-1}dG \ &= \ i(\,\Omega_Q \cdot Q + \Omega_S \cdot S + \Omega_P \cdot P + \Omega_Z \cdot Z + \Omega_D D + \Omega_K \cdot K \\
&\quad + \Omega_{\widetilde{Z}} \cdot \widetilde{Z} + \Omega_L \cdot L + \Omega_A A + \Omega_F \cdot F\,) \equiv i\Omega
\end{aligned}
\tag{3.7}
$$

where the notation basically follows the bosonic case and $\Omega_Q \cdot Q = \Omega_Q^\alpha Q_\alpha + \bar{\Omega}_{Q\dot\alpha}\bar{Q}^{\dot\alpha}$, etc. Once again, we explicitly present only few forms needed for our purpose

$$
\begin{aligned}
\Omega_Q^\alpha &= e^{\frac{1}{2}\phi}\,d\theta^\alpha + i\hat\omega_P^{\alpha\dot\alpha}\bar\psi_{\dot\alpha} + 2i\hat\omega_Z^{\alpha\beta}\psi_\beta\,, \\[4pt]
\Omega_S^\alpha &= d\psi^\alpha + \tfrac{1}{2}e^{\frac{1}{2}\phi}(\psi)^2 d\theta^\alpha - e^{\frac{1}{2}\phi}\psi^\alpha(\bar\psi d\bar\theta) - ie^{\frac{1}{2}\phi}d\bar\theta_{\dot\alpha}k^{\dot\alpha\alpha} \\
&\quad + 2ie^{\frac{1}{2}\phi}d\theta^\beta t_\beta^\alpha + \tfrac{1}{2}\psi^\alpha\hat\omega_D + 2i\hat\omega_L^{\beta\alpha}\psi_\beta - i\psi^\alpha\hat\omega_A + 2i\hat\omega_F^{\alpha\dot\alpha}\bar\psi_{\dot\alpha}\,, \\[4pt]
\Omega_P^{\alpha\dot\alpha} &= \hat\omega_P^{\alpha\dot\alpha}\,, \ \ \Omega_Z^{\alpha\beta} = \hat\omega_Z^{\alpha\beta}\,, \ \ \Omega_D = \hat\omega_D + e^{\frac{1}{2}\phi}d\theta^\alpha\psi_\alpha + e^{\frac{1}{2}\phi}d\bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha}\,.
\end{aligned}
\tag{3.8}
$$

The one-forms with 'hat' are obtained from the forms (3.5) via the replacements

$$dx^{\alpha\dot\alpha} \ \Rightarrow \ \Delta x^{\alpha\dot\alpha} = dx^{\alpha\dot\alpha} - i\left(\theta^\alpha d\bar\theta^{\dot\alpha} + \bar\theta^{\dot\alpha} d\theta^\alpha \right)\,,$$

$$dz^{\alpha\beta} \ \Rightarrow \ \Delta z^{\alpha\beta} = dz^{\alpha\beta} + i\theta^{(\alpha}d\theta^{\beta)}\,, \ \ \ d\bar{z}^{\dot\alpha\dot\beta} \ \Rightarrow \ \Delta \bar{z}^{\dot\alpha\dot\beta} = d\bar{z}^{\dot\alpha\dot\beta} + i\bar\theta^{(\dot\alpha}d\bar\theta^{\dot\beta)}\,. \tag{3.9}$$

Now let us show that the fermionic Goldstone superfields $\psi^\alpha(\widetilde{Y}), \bar{\psi}^{\dot\alpha}(\widetilde{Y})$, as well as the bosonic ones $k^{\alpha\dot\alpha}(\widetilde{Y})$, $t^{\alpha\beta}(\widetilde{Y})$ and $\bar{t}^{\dot\alpha\dot\beta}(\widetilde{Y})$, can be covariantly eliminated by imposing one basic inverse super-Higgs [15] constraint

$$\Omega_D = 0\,. \tag{3.10}$$

For bosonic Goldstone superfields this constraint yields

$$k_{\alpha\dot\alpha} = -e^{-\phi}\,\partial_{\alpha\dot\alpha}\phi\,, \ \ t_{\alpha\beta} = \frac{1}{2}e^{-\phi}\,\partial_{\alpha\beta}\phi\,, \ \ \bar{t}_{\dot\alpha\dot\beta} = \frac{1}{2}e^{-\phi}\,\partial_{\dot\alpha\dot\beta}\phi\,, \tag{3.11}$$

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[6]We define the contraction of two Weyl spinors in the standard way, $\psi \cdot \xi = \psi^\alpha\xi_\alpha$, $\bar\psi\bar\xi = \bar\psi_{\dot\alpha} \cdot \bar\xi^{\dot\alpha}$, $(\psi)^2 = \psi^\alpha\psi_\alpha$, also $x^2 = x^{\alpha\dot\alpha}x_{\alpha\dot\alpha}$, etc.

while for the fermionic ones we obtain

$$\psi_\alpha = -e^{-\frac{1}{2}\phi} D_\alpha \phi, \quad \bar{\psi}_{\dot\alpha} = -e^{-\frac{1}{2}\phi} \bar{D}_{\dot\alpha} \phi, \tag{3.12}$$

where

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar{\theta}^{\dot\beta}\partial_{\alpha\dot\beta} + i\theta^\beta\partial_{\alpha\beta}, \quad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\theta^\beta\partial_{\beta\dot\alpha} - i\bar{\theta}^{\dot\beta}\partial_{\dot\alpha\dot\beta}, \tag{3.13}$$

$$\{D_\alpha, \bar{D}_{\dot\alpha}\} = 2i\partial_{\alpha\dot\alpha}, \quad \{D_\alpha, D_\beta\} = 2i\partial_{\alpha\beta}, \quad \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 2i\partial_{\dot\alpha\dot\beta}. \tag{3.14}$$

Thus all Goldstone superfields have been expressed through the single basic scalar Goldstone superfield $\phi(\tilde{Y})$ associated with the dilatonic generator D. This superfield is the basic object of the nonlinear realization considered.

As the last topic of this section we present the transformation rules of the basic coset parameters under the Poincaré and conformal supersymmetries.

In our case all bosonic transformations are generated in the closure of the Poincaré supersymmetry transformations (left shifts with $Q_\alpha, \bar{Q}_{\dot\alpha}$) and transformations of the 'conformal' supersymmetry (left shifts with $S_\alpha, \bar{S}_{\dot\alpha}$). So it is enough to know those transformations which are induced by the left multiplication of the supercoset element (3.6) by an element $e^{i(a\cdot X)}$ with

$$(a \cdot X) = \epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot\alpha}\bar{Q}^{\dot\alpha} + \eta^\alpha S_\alpha + \bar{\eta}_{\dot\alpha}\bar{S}^{\dot\alpha}. \tag{3.15}$$

After straightforward computations using the $osp(1|8)$ structure relations we find

$$\delta x^{\alpha\dot\alpha} = i(\epsilon^\alpha\bar{\theta}^{\dot\alpha} + \bar{\epsilon}^{\dot\alpha}\theta^\alpha) + 2\eta_\beta z^{\beta\alpha}\bar{\theta}^{\dot\alpha} - 2\bar{\eta}_{\dot\beta}\bar{z}^{\dot\beta\dot\alpha}\theta^\alpha - \eta_\beta x^{\beta\dot\alpha}\theta^\alpha + \bar{\eta}_{\dot\beta}x^{\alpha\dot\beta}\bar{\theta}^{\dot\alpha},$$

$$\delta z^{\alpha\beta} = i\theta^{(\alpha}\epsilon^{\beta)} - 2\eta_\gamma z^{\gamma(\alpha}\theta^{\beta)} - \bar{\eta}_{\dot\alpha}\theta^{(\alpha}x^{\beta)\dot\alpha}, \quad \delta\bar{z}^{\dot\alpha\dot\beta} = i\bar{\theta}^{(\dot\alpha}\bar{\epsilon}^{\dot\beta)} + 2\bar{\eta}_{\dot\gamma}\bar{z}^{\dot\gamma(\dot\alpha}\bar{\theta}^{\dot\beta)} + \eta_\alpha x^{\alpha(\dot\alpha}\bar{\theta}^{\dot\beta)},$$

$$\delta\theta^\alpha = \epsilon^\alpha - 2i\eta_\beta z^{\beta\alpha} + \frac{1}{2}(\theta)^2\eta^\alpha + \bar{\eta}_{\dot\alpha}(\theta^\alpha\bar{\theta}^{\dot\alpha} - ix^{\alpha\dot\alpha}),$$

$$\delta\bar{\theta}^{\dot\alpha} = \bar{\epsilon}^{\dot\alpha} + 2i\bar{\eta}_{\dot\beta}\bar{z}^{\dot\beta\dot\alpha} + \frac{1}{2}(\bar{\theta})^2\bar{\eta}^{\dot\alpha} + \eta_\alpha(\theta^\alpha\bar{\theta}^{\dot\alpha} + ix^{\alpha\dot\alpha}), \tag{3.16}$$

$$\delta\phi = \phi'(\tilde{Y}') - \phi(\tilde{Y}) = -(\eta\theta + \bar{\eta}\bar{\theta}). \tag{3.17}$$

The transformation properties of the remaining Goldstone fields can be easily found using the inverse Higgs expressions (3.11), (3.12), the transformation law (3.17) and the transformation rules of the derivatives $\partial_{\alpha\dot\alpha}, \partial_{\alpha\beta}, \partial_{\dot\alpha\dot\beta}, D_\alpha, \bar{D}_{\dot\alpha}$, e.g.,

$$\delta D_\alpha = -(\theta_\alpha\eta^\beta - \eta_\alpha\theta^\beta)D_\beta - (\eta\theta)D_\alpha - 2(\eta_\alpha\bar{\theta}^{\dot\beta})\bar{D}_{\dot\beta},$$

$$\delta\bar{D}_{\dot\alpha} = (\bar{\theta}_{\dot\alpha}\bar{\eta}^{\dot\beta} - \bar{\eta}_{\dot\alpha}\bar{\theta}^{\dot\beta})\bar{D}_{\dot\beta} - (\bar{\eta}\bar{\theta})\bar{D}_{\dot\alpha} + 2(\bar{\eta}_{\dot\alpha}\theta^\beta)D_\beta. \tag{3.18}$$

4 Higher spin dynamics from Cartan forms

To see how the higher spin dynamics arises within the nonlinear realizations approach let us substitute the inverse Higgs expression (3.11), (3.12) for the Goldstone superfields into the covariant differentials of the superspace coordinates, i.e.

$\Omega_Q^\alpha, \bar\omega_Q^{\dot\alpha}, \Omega_P^{\alpha\dot\alpha}, \Omega_Z^{\alpha\beta}, \bar\Omega_Z^{\dot\alpha\dot\beta}$, and the covariant differentials of the fermionic Goldstone superfields $\Omega_S^\alpha, \bar\Omega_S^{\dot\alpha}$. As a consequence the fermionic part of Ω_S^α takes the simple form

$$\Omega_S^\alpha| = \Omega_Q^\beta \left\{ e^{-2\phi} D^{[\alpha} D_{\beta]} e^\phi \right\} + \bar\Omega_{\dot\beta Q} \left\{ e^{-2\phi} D^{[\alpha} \bar D^{\dot\beta]} e^\phi \right\}. \tag{4.1}$$

Then the desired equations for higher spins follow from the requirement that these projections (and their conjugates) vanish:

$$(D)^2 e^\phi = (\bar D)^2 e^\phi = 0, \quad \left[D^\alpha, \bar D^{\dot\alpha} \right] e^\phi = 0. \tag{4.2}$$

Eqs. (4.2) are recognized as the two-component spinor form of the equation suggested in [8] (for $\Phi = e^\phi$). The $OSp(1|8)$ covariance of (4.2) can be directly checked using (3.17), (3.18).

We observe that the superfield system (4.2) amounts to vanishing of *full* covariant *spinor* derivatives of the *spinor* Goldstone superfields $\psi^\alpha, \bar\psi_{\dot\alpha}$. Indeed, by definition

$$\Omega_S^\alpha| = \Omega_Q^\beta \nabla_\beta \psi^\alpha + \bar\Omega_{Q\dot\beta} \bar\nabla^{\dot\beta} \psi^\alpha, \tag{4.3}$$

where, with taking account of (3.11), (3.12),

$$\nabla_\beta \psi^\alpha = \delta_\beta^\alpha\, e^{-2\phi} (D)^2 e^\phi, \quad \bar\nabla^{\dot\beta} \psi^\alpha = \frac{1}{2} e^{-2\phi} \left[D^\alpha, \bar D^{\dot\beta} \right] e^\phi. \tag{4.4}$$

Let us present another form of eqs. (4.2) which is more suggestive. Prior to imposing the inverse Higgs constraints, the covariant derivatives of $\psi^\alpha, \bar\psi_{\dot\alpha}$ are as follows

$$\nabla_\beta \psi^\alpha = e^{-\frac{1}{2}\phi} D_\beta \psi^\alpha - \frac{1}{2} e^{-\frac{1}{2}\phi} \psi^\alpha D_\beta \phi + 2it_\beta^\alpha + \frac{1}{2} \delta_\beta^\alpha (\psi)^2,$$

$$\bar\nabla_{\dot\beta} \psi^\alpha = e^{-\frac{1}{2}\phi} \bar D_{\dot\beta} \psi^\alpha - \frac{1}{2} e^{-\frac{1}{2}\phi} \psi^\alpha \bar D_{\dot\beta} \phi - ik_{\dot\beta}^\alpha + \psi^\alpha \bar\psi_{\dot\beta}. \tag{4.5}$$

Then it is easy to show that the dynamical Eqs. (4.2), as well as the inverse Higgs expressions for the bosonic Goldstone superfields, can be derived from the following minimal set of equations

$$\nabla_\beta \psi^\alpha = 0, \quad \bar\nabla_{\dot\beta} \psi^\alpha = 0 \quad \text{and c.c.}, \tag{4.6}$$

$$\nabla_\beta \phi = 0, \quad \bar\nabla_{\dot\beta} \phi = 0, \tag{4.7}$$

where $\nabla_\beta \phi, \bar\nabla_{\dot\beta} \phi$ are covariant spinor projections of the Cartan form Ω_D:

$$\nabla_\beta \phi = \psi_\beta + e^{-\frac{1}{2}\phi} D_\beta \phi, \quad \bar\nabla_{\dot\beta} \phi = \bar\psi_{\dot\beta} + e^{-\frac{1}{2}\phi} \bar D_{\dot\beta} \phi. \tag{4.8}$$

Eqs. (4.7) express spinor Goldstone superfields through the superdilaton ϕ, then Eqs. (4.6) imply the expressions (3.11) for the bosonic Goldstone superfields and simultaneously yield the dynamical Eqs. (4.2). Actually, it is the traceless part of the first equation in (4.6) and the imaginary part $\sim (\bar\nabla_{\dot\alpha}\psi_\alpha + \nabla_\alpha \bar\psi_{\dot\alpha})$ of the second one which, together with (4.7), form a kinematical subset in the set (4.6), (4.7). The vanishing of the remaining covariant projections of the Cartan form Ω_D (associated with the forms $\Omega_P^{\alpha\dot\alpha}, \Omega_Z^{\alpha\beta}$ and $\bar\Omega_Z^{\dot\alpha\dot\beta}$) and, hence, of the whole Ω_D (Eq. (3.10)), is just a consequence of the Maurer-Cartan equations and the kinematical part of Eqs. (4.6), (4.7).

The formulation based on Eqs. (4.6), (4.7) is advantageous also because of its manifest $OSp(1|8)$ covariance which does not require any explicit checks. Indeed, all $OSp(1|8)$ transformations of the full covariant derivatives have the form of induced transformations of the stability subgroup $SL(4, R)$ acting on the spinor indices. The covariance under the $GL(2, C)$ transformations is evident and one should only be convinced of the covariance under the transformations generated by $F_{\alpha\dot\alpha}, \bar F_{\alpha\dot\alpha}$. From the general transformation of the Cartan form (3.7) it is easy to deduce the transformations of the spinor covariant derivatives of the involved Goldstone superfields

$$\delta(\nabla_\alpha \phi) = -2i\bar\lambda_{\alpha\dot\alpha}\bar\nabla^{\dot\alpha}\phi\,, \quad \delta(\bar\nabla_{\dot\alpha}\phi) = -2i\lambda_{\dot\alpha}^\alpha \nabla_\alpha\phi\,,$$

$$\delta(\nabla_\beta \psi^\alpha) = 2i\bar\lambda^{\alpha\dot\alpha}\,\nabla_\beta\bar\psi_{\dot\alpha} - 2i\bar\lambda_{\beta\dot\alpha}\,\bar\nabla^{\dot\alpha}\psi^\alpha\,,$$

$$\delta(\bar\nabla_{\dot\alpha} \psi^\beta) = 2i\bar\lambda^{\beta\dot\beta}\,\bar\nabla_{\dot\alpha}\bar\psi_{\dot\beta} - 2i\lambda_{\dot\alpha}^\gamma\,\nabla_\gamma\psi^\beta\,. \tag{4.9}$$

The full $OSp(1|8)$ covariance of the system (4.6), (4.7) is therefore obvious. In the manifestly $SL(4, R)$ covariant notation it just reads

$$\nabla_{\hat\alpha}\psi^{\hat\beta} = 0\,, \quad \nabla_{\hat\alpha}\phi = 0\,. \tag{4.10}$$

The geometric meaning of Eqs. (4.6), (4.7) can be further clarified, using Maurer-Cartan equations. Let us split the general $osp(1|8)$ superalgebra valued Cartan form (3.7) into the parts $\Omega_\perp$ and $\Omega_=$, spanned, respectively, by the coset generators and those of the stability subgroup $SL(4, R)$:

$$\Omega = \Omega_\perp + \Omega_=\,, \quad d \wedge \Omega + i\Omega \wedge \Omega = 0 \;\Rightarrow$$

$$\mathcal{T} \equiv d \wedge \Omega_\perp + i\Omega_= \wedge \Omega_\perp + i\Omega_\perp \wedge \Omega_= = -i\,(\Omega_\perp \wedge \Omega_\perp)\,|_\perp\,, \tag{4.11}$$

$$\mathcal{R} \equiv d \wedge \Omega_= + i\Omega_= \wedge \Omega_= = -i\,(\Omega_\perp \wedge \Omega_\perp)\,|_=\,, \tag{4.12}$$

where $|_\perp$ and $|_=$ denote the restriction to the suitable coset and stability subgroup generators. The supercoset (1.2) is not symmetric, therefore both the torsion and curvature two-superforms $\mathcal{T}$ and $\mathcal{R}$ are non-vanishing. Since the parameters of the coset are separated into the coordinates of the superspace $R^{(10|4)} = (Y^{(\hat\alpha\hat\beta)}, \theta^{\hat\alpha}) \equiv \widetilde{Y}^M$ and Goldstone superfields given on $\widetilde{Y}^M$, we actually deal with the $R^{(10|4)}$-pullbacks of $\mathcal{T}$ and $\mathcal{R}$. The supertorsion and supercurvature tensors can be defined as

$$\mathcal{T} = \mathcal{T}_{MN}\Omega^M \wedge \Omega^N\,, \quad \mathcal{R} = \mathcal{R}_{MN}\Omega^M \wedge \Omega^N\,, \tag{4.13}$$

where

$$\Omega^M = \left(\Omega_P^{\alpha\dot\beta}, \Omega_Z^{\alpha\beta}, \bar\Omega_Z^{\dot\alpha\dot\beta}, \Omega_Q^\alpha, \bar\Omega_Q^{\dot\alpha}\right)\,. \tag{4.14}$$

The considerations based on the $osp(1|8)$ anticommutation relations (2.4a) and the fact that the $sl(4, R)$ generators appear only in the mixed anticommutators (between S and Q generators) show that Eqs. (4.6) give rise to the vanishing of the supercurvature tensor components in the pure Grassmann directions

$$\mathcal{R}_{\hat\alpha\hat\beta} = \left(\mathcal{R}_{\alpha\beta}, \mathcal{R}_{\dot\alpha\dot\beta}, \mathcal{R}_{\alpha\dot\beta}\right) = 0\,, \tag{4.15}$$

while (4.7) amount to the vanishing of certain supertorsion components. Thus these equations are equivalent to the particular zero-curvature (and zero-torsion) conditions.

The geometric nature of these simple dynamical conditions deserves further study. In this connection, it is worth to note that the vanishing of the covariant spinor world-supersurface projections of the *vector* Cartan form of the target superspace is the basic postulate of the embedding approach to superbranes (see [22] and refs. therein). Also, the vanishing of the world-supervolume spinor covariant derivative of the spinor Goldstone superfield is the dynamical equation of $N = 1, 4D$ supermembrane in the approach based on the concept of partial breaking of global supersymmetry (PBGS) [23]. One more relevant analogy is suggested by the fact that the superfield equations of motion of some integrable supersymmetric $2D$ systems can be reformulated as a dynamical inverse Higgs phenomenon (see e.g. [24]).

The manifestly $OSp(1|8)$ covariant formulation and the transformation laws (4.9) can provide the convenient starting point for the search of the appropriate manifestly covariant action and possible extension of the equations (4.6), (4.7) to the case with interaction. We also note that the system (4.6), (4.7) contains only one derivative (spinor or bosonic) and so appears similar to the 'unfolded' form of the equations for higher spin fields characterized by the same feature (see [7, 3]). Perhaps it could be put precisely in this form by adding some supplementary equations which possibly are satisfied as a consequence of (4.6), (4.7).[7]

5 Tensorial chiral superspace: a proper setting for higher spin $N = 1$ supergravity?

An important problem for further study is the application of our approach to higher spin extensions of $N = 1$, $4D$ supergravity. In the standard conformal $N = 1$ supergravity a purely geometric approach has been proposed by Ogievetsky and Sokatchev [16]. The underlying $N = 1$ supergravity gauge group in this approach is a group of general diffeomorphisms of chiral $N = 1$ superspace $C^{(4|2)} = (x_L^m, \theta_L^\alpha)$, which exposes the fundamental role of the principle of preserving $N = 1$ chiral representations in $N = 1$ supergravity. The question arises whether an analog of this principle can be formulated for higher-spin generalization of $N = 1$ supergravity. From the analysis of the full set of the (anti)commutation relations of the superalgebra $osp(1|8)$ it follows that the minimal analog of $C^{(4|2)}$ is the coset spanned by the following generators

$$\left(P_{\alpha\dot{\alpha}}, Z_{\alpha\beta}, F_{\beta\dot{\beta}}, Q_\alpha \right), \tag{5.1}$$

i.e. it contains only one holomorphic half of the tensorial central charges and, in addition, the complex generator $F_{\beta\dot{\beta}}$. It is easy to check that the rest of the $osp(1|8)$ generators form a complex non-self-conjugated subalgebra, so the set of the coset parameters associated with the generators (5.1), i.e.

$$C^{(11|2)} = (x_L^{\alpha\dot{\beta}}, z_L^{\alpha\beta}, f_L^{\alpha\dot{\beta}}, \theta_L^\alpha) \equiv (Y_L), \tag{5.2}$$

is closed under the left action of the supergroup $OSp(1|8)$ and provides a natural generalization of $C^{(4|2)}$. Note that $f_L^{\alpha\dot{\beta}}$ yield a holomorphic parametrization of the coset $SL(4, R)/GL(2, C)$ and so are a sort of harmonic variables. Thus $C^{(11|2)}$ can be also treated as an analytic subspace of the 'harmonic superspace' $R^{(10|4)} \times \frac{SL(4,R)}{GL(2,C)}$.

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[7] E.I. thanks M. Vasiliev for suggesting this possibility.

The precise realization of $OSp(1|8)$ in the coset manifold (5.2) can be straightforwardly found and it will be discussed in a future publication. Here we only give how the coordinates (5.2) are related to the $R^{(10|4)}$ ones:

$$\theta_L^\alpha = \theta^\alpha - 2if_L^{\alpha\dot\alpha}\bar\theta_{\dot\alpha}, \quad x_L^{\alpha\dot\alpha} = x^{\alpha\dot\alpha} - i\theta^\alpha\bar\theta^{\dot\alpha} - 4if_{L\ \dot\alpha}^\alpha\bar{z}^{\dot\alpha\dot\beta} - (\bar\theta)^2 f_L^{\alpha\dot\beta},$$

$$z_L^{\alpha\beta} = z^{\alpha\beta} + 4f_{L\ \dot\beta}^\alpha f_{L\ \dot\gamma}^\beta\bar{z}^{\dot\beta\dot\gamma} - 2\theta^{(\alpha}f_L^{\beta)\dot\beta}\bar\theta_{\dot\beta}, \tag{5.3}$$

and how $f_L^{\alpha\dot\alpha}$ is transformed under conformal supersymmetry

$$\delta f_L^{\alpha\dot\alpha} = i\bar\eta^{\dot\alpha}\theta_L^\alpha + 2\eta^{(\alpha}\theta_L^{\beta)}f_{L\beta}^{\dot\alpha} + (\eta\cdot\theta_L)f_L^{\alpha\dot\alpha}. \tag{5.4}$$

It is interesting to inquire whether some higher-spin dynamics can be associated with superfields given on (5.2) as an alternative to eqs. (4.2) and what is the theory enjoying invariance under general diffeomorphisms of $C^{(11|2)}$ (the higher spin analog of $N=1, 4D$ conformal supergravity in the Ogievetsky-Sokatchev formulation?) Leaving the complete analysis of these issues for the future, we give here the $OSp(1|8)$ invariant tensorial superspace analogs of the standard kinetic and potential terms of $N=1, 4D$ chiral superfields. Defining the integration measures in the central and chiral superspaces

$$\mu = d^4x\, d^6z\, d^4\theta\, d^4\bar\theta\, f_L\, d^4 f_R, \quad \mu_L = d^4x_l\, d^3z_L\, d^2\theta_L\, d^4 f_L \quad \left(f_R^{\dot\alpha\alpha} \equiv \overline{(f_L^{\alpha\dot\alpha})}\right), \tag{5.5}$$

one can show that they transform as

$$\delta\mu = 8\left[(\eta\cdot\theta) + (\bar\eta\cdot\bar\theta) + 2i\left(\eta_\alpha\bar\theta_{\dot\alpha}f_L^{\alpha\dot\alpha} + \bar\eta_{\dot\alpha}\theta_\alpha f_R^{\alpha\dot\alpha}\right)\right]\mu, \quad \delta\mu_L = 12(\eta\cdot\theta_L)\mu_L. \tag{5.6}$$

Now the $OSp(1|8)$ invariant kinetic term of the superfield $\Phi(Y_L)$ is uniquely defined to be

$$S_{kin} \sim \int \mu\, \Phi(Y_L)\bar\Phi(Y_R), \quad \left(Y_R = \overline{(Y_L)}\right) \tag{5.7}$$

where Φ is transformed as

$$\delta\Phi = -8(\eta\cdot\theta_L)\Phi. \tag{5.8}$$

The $OSp(1|8)$ invariant potential term of $\Phi(Y_L)$ is also unique

$$S_{pot} = \int \mu_L\Phi^{\frac{3}{2}} + \text{c.c.}. \tag{5.9}$$

The component contents of these actions and their relation to the higher spin theory will be analyzed elsewhere. It still remains to give the precise meaning to the integration over the auxiliary tensorial and $SL(4,R)/GL(2,C)$ variables.

6 Further developments

In this paper we did show that nonlinear realizations of the generalized $4D$ superconformal group $OSp(1|8)$ provide a natural framework for treating massless higher spins in the 10-dimensional space with tensorial coordinates. The superfield equation encompassing all free equations for integer and half-integer spins [6]-[8] was derived on a geometric ground as the condition of vanishing of the covariant spinor projections of

some basic Cartan one-forms on the supercoset (1.2). Via Maurer-Cartan equations, this dynamical equation implies vanishing of the supercurvature tensor along the pure odd directions in the tensorial superspace $R^{(10|4)}$. The basic scalar superfield has an intrinsic origin in the considered nonlinear realizations framework as the coset parameter (Goldstone superfield) associated with the spontaneously broken dilatations. We also generalized the important notion of chirality to the case of tensorial superspaces (with the ultimate aim to apply this to higher spin supergravity) and constructed first examples of the $OSp(1|8)$ invariant off-shell actions for the tensorial chiral superfields. These actions are expected to describe a non-trivial self-interaction of higher spins.

Besides suggesting these new insights into the theory of the massless higher spins, the nonlinear realization approach offers some other possibilities which we are planning to study elsewhere. Below we list some of them.

i) AdS higher spin theories. In order to gain massive higher spin theories in the coset framework, e.g. on the AdS background, one should pass to the curved standard and tensorial translations which belong to the following closed set of generators. For example, for the bosonic $Sp(8)$ subgroup of $OSp(1|8)$ we get

$$\hat{P}_{\alpha\dot\alpha} = P_{\alpha\dot\alpha} + m^2 K_{\alpha\dot\alpha}\,,\ \hat{Z}_{\alpha\beta} = Z_{\alpha\beta} + m^2 \widetilde{Z}_{\alpha\beta}\,,\ \hat{\overline{Z}}_{\dot\alpha\dot\beta} = \overline{Z}_{\dot\alpha\dot\beta} + m^2 \widetilde{\overline{Z}}_{\dot\alpha\dot\beta}\,,$$

$$X_{\alpha\dot\beta} = F_{\alpha\dot\beta} + \bar{F}_{\alpha\dot\beta}\,,\quad L_{\alpha\beta},\ \overline{L}_{\dot\alpha\dot\beta}\,, \tag{6.1}$$

where m is a contraction parameter having the dimension of mass (inverse AdS radius). Introducing the coordinates just for the curved translation generators,[8] constructing the corresponding Cartan forms and imposing on them the appropriate covariant dynamical conditions, we should obtain the counterpart of Eqs. (4.2) for free higher-spin fields on AdS_4 background [20, 8]. These equations, as they stand, are presumably related to the presented here massless ones (4.2) via the generalized Weyl transformation defined in [19, 20]. Interaction terms should break full $OSp(1|8)$ symmetry and, hence, the conformal equivalence of the AdS_4 and flat cases.

ii) Tensorial analog of AdS_5 branes. One can consider a possible relation of the nonlinear realizations of $OSp(1|8)$ to some AdS brane-like objects with the tensorial space as the worldvolume. We shall limit our discussion to the bosonic group $Sp(8)$. Of relevance for us will be a subgroup of $Sp(8)$ generated by the following set of generators

$$\hat{P}_{\alpha\dot\alpha} = P_{\alpha\dot\alpha} - m^2 K_{\alpha\dot\alpha}\,,\ \hat{Z}_{\alpha\beta} = Z_{\alpha\beta} - m^2 \widetilde{Z}_{\alpha\beta}\,,\ \hat{\overline{Z}}_{\dot\alpha\dot\beta} = \overline{Z}_{\dot\alpha\dot\beta} - m^2 \widetilde{\overline{Z}}_{\dot\alpha\dot\beta}\,,$$

$$X_{\alpha\dot\beta} = F_{\alpha\dot\beta} + \bar{F}_{\alpha\dot\beta}\,,\quad L_{\alpha\beta},\ \overline{L}_{\dot\alpha\dot\beta}\,. \tag{6.2}$$

The group generated by (6.2) contains $SO(1,4) \propto (P_{\alpha\dot\beta} - m^2 K_{\alpha\dot\beta}, L_{\alpha\beta}, \bar{L}_{\dot\alpha\dot\beta})$ as a subgroup and describes an extension of $SO(1,4)$ by the tensorial nonlinear translations generated by $\hat{Z}_{\alpha\beta}$, $\hat{\overline{Z}}_{\dot\alpha\dot\beta}$ and $X_{\alpha\dot\beta}$.

In $4D$ case we can parametrize the AdS_5 coset $\frac{SO(2,4)}{SO(1,4)}$ by the coordinates $x^{\alpha\dot\beta}$ and dilaton ϕ [25], and obtain the description of AdS_5 3-brane [26]. In the $Sp(8)$ case the true analog of AdS_5 is just the coset of $Sp(8)$ over the subgroup generated by (6.2). This $Sp(8)$ coset manifold contains AdS_5 as a subspace, but it is much larger, because the full set of the coset generators is the following

$$P_{\alpha\dot\alpha}, Z_{\alpha\beta}, \bar{Z}_{\dot\alpha\dot\beta}, D, A, G_{\gamma\dot\gamma} = i(F_{\gamma\dot\gamma} - \bar{F}_{\gamma\dot\gamma})\,. \tag{6.3}$$

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[8]The set (6.1) is a sum of two isomorphic algebras $sp(4) \sim o(2,3)$. The coordinates associated with the curved translations parametrize the symmetric coset $Sp(4) \times Sp(4)/Sp(4)_{diag}$ (see e.g. [20]).

It contains

- 10-dimensional extended space-time manifold $R^{10} = (x^{\alpha\dot\alpha}, z^{\alpha\beta}, \bar{z}^{\dot\alpha\dot\beta})$ associated with the generators $(P_{\alpha\dot\alpha}, Z_{\alpha\beta}, \bar{Z}_{\dot\alpha\dot\beta})$. It is supplemented by dilaton.
- additional 5 dimensions generated by A and $G_{\alpha\dot\beta}$.

We see that the $Sp(8)$ analog of the AdS_5 3-brane with R^{10} as the worldvolume should involve besides Goldstone dilation field also further five transverse coordinates: one pseudoscalar coordinate generated by A and a real vector one associated with $G_{\alpha\dot\alpha}$. It is tempting to describe such an exotic brane-like object (and its superextension related to $OSp(1|8)$) and to see how it is related to the higher-spin theories.

iii) Towards higher dimensions. In this work we considered the $4D$ case for simplicity. The generalization of our approach to $D > 4$ implies the application of appropriate nonlinear coset realizations of the generalized $11 \geq D > 4$ superconformal algebras described by suitable real forms of $OSp(1|2^k)$ ($6 \geq k > 3$). Because the higher spin theories in diverse dimensions are intensively studied (see [3] and refs. therein, as well as [8] - in the context of tensorial superspaces), such generalization should be also investigated.

Acknowledgments

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